

Appendix B

Hyperbolic Functions

The following are some useful relations involving hyperbolic trigonometric functions:

Definitions:

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad \operatorname{csch}(x) = \frac{1}{\sinh(x)}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \operatorname{sech}(x) = \frac{1}{\cosh(x)}$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad \operatorname{ctnh}(x) = \frac{1}{\tanh(x)}$$

Useful relations:

$$\sinh(-x) = -\sinh(x)$$

$$\cosh(-x) = \cosh(x)$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

Derivatives:

$$\frac{d}{dx} \sinh(x) = \cosh(x) \qquad \frac{d}{dx} \operatorname{csch}(x) = -\operatorname{csch}(x) \operatorname{ctnh}(x)$$

$$\frac{d}{dx} \cosh(x) = \sinh(x) \qquad \frac{d}{dx} \operatorname{sech}(x) = -\operatorname{sech}(x) \operatorname{tanh}(x)$$

$$\frac{d}{dx} \tanh(x) = \operatorname{sech}^2(x) \qquad \frac{d}{dx} \operatorname{ctnh}(x) = -\operatorname{csch}^2(x)$$

Expansions for small x:

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$$

$$\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$$

$$\tanh(x) = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \dots$$

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