Experiment 2 Rotational Dynamics

Background

You may find it useful to review chapters from your introductory physics textbook dealing with rotational motion, torque and angular momentum.

Torque is defined through the equation

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad (1)$$

In the case of a force applied at right angles to the lever arm, which is the case when a string is wound around a pulley, the magnitude of the torque is just

$$\tau = rF, \quad (2)$$

where r is the radius of the pulley and F is the tension in the string.

Moment of inertia is a measure of the resistance of an object to changes in its rotational motion. The value of the moment of inertia depends on the axis of rotation and on the distribution of mass in the object. For some simple situations we have

$I=mr^2$	A point mass m a distance r from the axis of rotation
$I=mR^2$	A thin hoop or ring of mass m and radius R rotated about an axis perpendicular to it and passing through its center
$I=(1/2)mR^2$	A uniform solid disk of mass m and radius R rotated about an axis perpendicular to it and passing through its center
$I=(1/12)mL^2$	A thin rod of mass m and length L rotated about an axis perpendicular to it and passing through its center

Moments of inertia of more complex objects can sometimes be found by breaking the object down into components and adding up the individual contributions to the moment of inertia.

The moment of inertia relates torque to angular acceleration:

$$\tau_{net} = I\alpha \qquad (3)$$

The moment of inertia also shows up when calculating angular momentum:

$L = I\omega \qquad (4)$

In the absence of external torques on a system, the angular momentum is conserved. This means that in a collision between rotating objects, to a good approximation the total angular momentum of the objects just prior to the collision is equal to the total angular momentum of the objects just after the collision.

Procedure

Part 1: Varying torque

1. You will be using the same rotational dynamics apparatus you used for the first experiment. Initially it should be set up as before, with the rotating platform mounted and a string wrapped around one of the pulleys. Adjust the sliding masses so that they are both centered at 15 cm from the center. In this lab, you need to adjust these weights with accuracy. A hole through one edge of the block allows you to see through to the scale below. Reminder: DO NOT OVERTIGHTEN THE SCREWS.

2. Wind the string on the uppermost platform pulley and suspend a 50 g mass from it. Starting from rest, time the rotating platform as it executes 5 full revolutions. Repeat for a total of three measurements.

3. Change the hanging mass to 70 g by hooking a 20g mass to the bottom of the 50 g mass, and repeat step 2.

4. Change the hanging mass to 100 g and repeat step 2.

5. Now repeat steps 2-4 for the middle platform pulley and then the bottom platform pulley. In the end, you should have taken 27 measurements.

6. Detach the rotating platform by loosening the black set screw underneath the platform. Measure the length of the platform, and then measure the mass of the rotating platform and the two adjustable masses *separately*. (The black base at the center of the rotating platform has a mass of 70.9g. It contributes a negligible amount to the moment of inertia of the platform since that mass is concentrated near the center of the platform. When you go to calculate the moment of inertia of the platform, subtract this mass from the total mass you measured on the scale.) Use calipers to measure the diameter of the three platform pulleys and remember to convert these numbers to radii for your calculations. Make sure you are measuring the radius of the pulley out to the point at which the thread makes contact with the pulley (not past that to the lip of the pulley).

Part 2: Varying moment of inertia

1. Reattach the rotating platform to the shaft, being careful to line up the set screw with the flat side of the shaft. Wind the string around the middle platform pulley. Adjust the sliding masses so that they are each centered 3cm from the middle of the rotating platform. Hang a 50 g mass from the thread and time the platform through 5 revolutions starting from rest. Repeat to obtain three measurements.

2. Repeat step 1, moving both masses successively through the following positions : 8cm, 13cm, 18cm, 23cm. In each case, these positions refer to the center of each mass. In the end, you will have taken 15 measurements.

Part 3: Conservation of angular momentum

1. Loosen the set screw underneath the rotating platform and slide the platform off, setting it aside in a safe location. Wind the thread completely around the pulley and use a small piece of tape to secure it to the pulley to keep it from getting tangled. You will not need the thread for this part of the experiment.

2. You should see a nearly solid black disk with a circular groove in it. Mount it where the rotating platform was. If this disk does not slide onto the shaft smoothly, you do not have the flat side of the hole lined up properly with the flat side of the shaft. You should also see a black hollow cylinder which is made to fit in the groove. Spin the solid disk. Hold the hollow cylinder just above the groove on the disk and practice dropping the cylinder onto the disk while the disk is rotating. Make sure you let go of the cylinder completely before it makes contact with the rotating disk.

3. Remove the hollow cylinder from the disk. Give the disk a spin so that it is taking roughly 0.5s to complete a revolution. Time the disk for *ten* consecutive revolutions. As soon as possible, drop the cylinder onto the groove and then immediately time another 10 revolutions. Repeat this process for a total of five pairs of measurements. These times will allow you to calculate the angular velocity before and after the collision. It is important to allow as little time as possible to elapse between the time measurements and the collision in order to minimize the effect of friction.

4. Remove both the hollow cylinder and the solid black disk. Remove the tape that was securing the thread and remount the rotating platform. This is a "lab courtesy" step which gets the apparatus ready for the next group.

5. Measure the mass of the hollow cylinder and the solid disk. Note to measure masses greater than 500g on the balance beam, you need to add one or more tan masses to the tiny knobs at the right end of the balance scale. These tan masses are labeled 500g and 1000g, and they represent the effective leverage they apply to the scale, not their true masses. The bottom line is that if for instance you want to measure something with a 650g mass, you will need a tan mass labeled 500g hooked onto the knob (onto the smooth part, not onto the threads) and the scale should balance when the slides are positioned to 100+50 g.

6. Measure the diameter of the solid disk and of the hollow cylinder. One way to do this accurately is to first measure the circumference by measuring the length of a thread that is wrapped around the outside. The radius is this length divided by 2π. Be sure to return the solid disk and the hollow cylinder to the end of the table with the rest of the apparatus when you are done.

Data Analysis

This section will help guide you through writing up the Results section of your report. A complete Results section should show all the raw data (in tabular form when you have a lot), calculated results with uncertainties, sample calculations of each type, and a discussion of the results. Make sure that this section includes sufficient text to be readable. That is, do not just show a data table but introduce it with a sentence or two indicating what data you are showing (e.g., "The following data was acquired to test the principle of conservation of angular momentum."). Put another way, the Results section is not just a collection of numbers and calculations; it is **TEXT** interspersed with data and calculations. As always, consult the sample lab report online for further information and keep in mind the comment on significant digits in the Experiment 1 write-up.

Part 1

For each of the pulley and mass combinations, calculate the angular acceleration using the first approach from experiment 1 (that is, the non-graphical approach). Use equation (2) to calculate the applied torque, taking the tension in the string to be approximately mg. Using a spreadsheet program, plot the angular acceleration (horizontal axis) vs. the applied torque (vertical axis) for all 9 mass/pulley combinations. If we have correctly analyzed the data and equation (3) is correct, the result should be a straight line passing through the origin. Add a trendline showing a linear fit. Under options, *do not* specify that it passes through the origin but do have it display the equation. The best-fit equation will likely not pass through the origin. As you discuss your results in this section, discuss the physics behind this non-zero intercept and discuss what the slope of the trend line represents. Remembering that both the calculated torque and the calculated acceleration have uncertainties, sketch those in as error bars on your printed plot or figure out how to get the spreadsheet to do that (this is possible in Excel—highlight the data series and then pull up the format data series menu). Make sure the final printed plot has all appropriate labeling.

Part 2

Knowing the mass and length of the rotating platform, you can estimate its moment of inertia if you treat it as a thin rod. Add to this the contributions from the two adjustable masses, treating them like point masses. You probably want to express everything in kilograms and meters. Calculate the approximate applied torque as above and then for each mass position you can predict an angular acceleration

$$\alpha_{pred} = \frac{\tau_{appl}}{I}$$

Compare this value to the measured angular acceleration, calculated the same way you did in Part 1. Each of these accelerations should have an uncertainty associated with it. If the uncertainty intervals overlap (e.g., if α_{pred}=2.3+/-0.4 and α_{meas}=2.1+/-0.3), then you have verified the theoretical prediction to within the uncertainties in your experiment. If the uncertainty intervals do not overlap (e.g., 2.3+/-0.4 and 1.7+/-0.1), then you have not verified the theoretical prediction. Among the possible reasons are (1) you underestimated the uncertainties; (2)the theory has been incorrectly applied because a significant effect was overlooked (such as friction); and (3) the theory is incorrect. Your discussion of the results should include your analysis about whether the uncertainty intervals overlap or not. You can postpone conclusions from these observations to the Conclusions section (which, fortunately for you, is not required for this lab).

Part 3

Treating the solid disk as uniform and the hollow cylinder as thin, calculate their moments of inertial. We expect

$$L_i = L_f$$

$$I_{disk}\omega_i = \left(I_{disk} + I_{cylinder}\right)\omega_f$$

$$\frac{\omega_f}{\omega_i} = \frac{I_{disk}}{I_{disk} + I_{cylinder}}$$

$$\frac{\Delta\theta / \Delta t_f}{\Delta\theta / \Delta t_i} = \frac{I_{disk}}{I_{disk} + I_{cylinder}}$$

$$\frac{\Delta t_i}{\Delta t_f} = \frac{I_{disk}}{I_{disk} + I_{cylinder}}$$

The ratio on the right hand side can now be calculated and should be the same for each trial. The ratio on the left hand side can be calculated separately for each trial. Note that is does *not* make sense to average the Δt's first since you do not expect them to be the same for each trial. Furthermore, since angular momentum should be conserved separately for each trial, we expect agreement in each trial to within expected uncertainties in this last equation. Your analysis should include a table showing the calculated values of the left hand side of the last equation compared to the predicted value, and an indication for each about whether they are within expected uncertainties. In Parts 1 and 2 you developed a good feel for expected uncertainties when timing rotational motion. You can use an estimate from those numbers for your uncertainty in Δt here.

You should also estimate uncertainties in the mass and the radius for your moment of inertia calculations.

What is due?

For this lab, you should turn in a title page and the Results section. The title page should include the name of your lab partner, the date you turned the report in, and an Abstract. The Abstract should describe the nature of the experiment in a few sentences and briefly summarize the results. This is not a mystery novel you are writing—it is expected that you reveal the essential results in the Abstract. **See the introductory material posted on the web site for a sample lab report (showing all sections, not just those required for this lab).** Check this every time you are learning how to write a new section. About half the points that are lost by students on these first few reports typically arise because they did not read the introductory section outlining expectations for the report and they did not consult the sample report.

Grading comment

Pay careful attention to the way sections of lab reports are graded (see the introductory material posted on the web site). In particular, the "error analysis multiplier" is a grading device designed to ensure that you take error analysis seriously. Your score on all sections related to results is multiplied by a number ranging from zero to one. The multiplier will be 1.0 if you do a credible (not necessarily perfect) job of estimating uncertainties and other sources of error, and carrying these estimates through numerically in all of your calculations. If you skip this, your error analysis multiplier will be 0, meaning you will get no credit for any of the relevant sections.