

Chapter 8: Torque and Angular Momentum

A Summary of main points of chapter

8.1 Rotational kinetic energy and rotational inertia

A rotating object has kinetic energy, since it is moving; however, different parts of it are moving with different speeds. It is simpler to write the kinetic energy in terms of the rotational speed (which is the same for all parts of the object, if it is rigid) and the rotational inertia. Finding the rotational inertia can be done simply for objects consisting of a small number of particles; for extended symmetrical objects, tables are available. (In general, calculating the rotational inertia for an extended object requires the use of calculus). The rotational inertia of an object depends not only on its shape but also on the location of the axis of rotation; this is because, for the same rotational speed, the linear speed of different parts of the object will depend on how far from the axis of rotation they are.

8.2 Torque

In the absence of appropriately directed external forces, a rotating object will continue rotating at the same rate as long as its rotational inertia does not change. In order to start or stop something rotating, it takes a force, but the force must be in a certain direction (not directly towards or away from the axis of rotation) and in a certain place (displaced from the axis of rotation) in order for rotation to result. The quantity that results in rotation is called torque; it is analogous to force in linear dynamics. A door provides a good example of torque: when we open a door, we instinctively push on the side away from the hinges (that is, as far away from the axis of rotation as possible), and perpendicular to the door's surface (that is, perpendicular to the direction towards the axis of rotation). This gives us the maximum torque for a given force. If we were to exert the same force, but towards the axis of rotation, or close to the hinges, much less (or even no) torque would result. By convention, we take the positive direction to be counterclockwise, so that a positive torque is one which would start a motionless object rotating counterclockwise; only the component of the torque perpendicular to the direction towards the axis of rotation contributes to the torque.

Center of gravity: The gravitational force on an object acts throughout the object, but in terms of the total torque, it can be considered to act at the center of gravity of the object. As long as the gravitational field is uniform throughout the object, the center of gravity is the same as the center of mass.

8.3 Work done by a torque

Since a torque is created by a force, a torque can do work just as a force can do work. Just as work done by a force is force times distance, work done by a torque is torque times angular displacement.

8.4 Equilibrium revisited

It is possible for an object to have no net force on it but still to have a net torque; in this case it will have an angular acceleration (resulting in rotation) but no linear acceleration (so its center of mass will not be moving). In order to be in equilibrium, not only must there be no net force on an object but there must be no net torque as well.

Distributed forces: The gravitational force is not the only one that does not act at a single point; the normal force and frictional forces are also distributed (along a surface rather than throughout a volume). For the normal force, if an object is about to topple, it will rotate around one line of contact, which is the axis of rotation. The normal force can then be considered to act through that line.

8.5 Equilibrium in the human body

When muscles contract, they pull on tendons, which pull on bones (muscles can pull but not push). If a limb is in equilibrium, it must have no net force or torque on it; the forces exerted by the tendons act at the attachment points.

Heavy lifting: If you bend over to pick up a heavy object, there will be a large torque on your back, in order to exert the needed force on the object. If the back is kept vertical and the knees bent instead, the torque on the back is minimized.

8.6 Rotational form of Newton's second law

The rotational form of Newton's second law tells us what angular acceleration results when a torque is exerted on an object; the net torque is equal to the rotational inertia multiplied by the angular acceleration.

8.7 The dynamics of rolling objects

If an object is rolling without slipping, its linear speed is equal to its rotational speed times its radius. Its total kinetic energy will be the sum of its translational and rotational kinetic energies.

8.8 Angular momentum

Just as the force can be written as the rate of change of the momentum, the torque can be written as the rate of change of angular momentum, where the angular momentum is equal to the rotational inertia multiplied by the angular speed. In the absence of external torques, the angular momentum is conserved.

Angular momentum in planetary orbits: Kepler's second law (that equal areas are swept out in equal times) is a consequence of the conservation of angular momentum.

8.9 The vector nature of angular momentum

The angular momentum is described as a vector along the axis of rotation with its direction determined by the right-hand rule. Not only the magnitude but also the direction of the angular momentum is unchanged in the absence of external torques.

A classic demonstration: If you hold a spinning bicycle wheel while standing on a platform that is free to rotate, you can demonstrate the conservation of the angular momentum of the entire system consisting of you, the platform, and the wheel while you move the wheel around.